MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2022/2023 (TERM 1D 2220)

TMA1101 – CALCULUS

(All sections / Groups)

08 JULY 2023
9.00 a.m. - 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **15 pages** with **5 Questions**, excluding this cover page.
2. Answer all **FIVE** questions. The distribution of the marks for each question is given on the right side of the margin.
3. Please write all your answers in this Question Booklet. Show all necessary working steps in each solution.
4. **No calculator is allowed.**

QUESTION 1 [10 MARKS]

a) Let $f(x)=\dfrac{1}{x^2-4}$.

i) State the domain of $f(x)$.

ii) Determine whether the function $f(x)$ is odd, even or neither. Justify your answer.

[3 marks]

Continued ...

b) Evaluate the following limits:
[***You must show at least one intermediate step where*** $\lim_{x\to c}$ ***is still needed.***]

(i) $\lim_{x\to 1}\left(\frac{x^2-1}{2x^2-x-1}\right)$ (ii) $\lim_{x\to\infty}\left(\frac{3x}{\sqrt{4x^2+x}}\right)$

[3 marks]

Continued ...

c) Given $f(x)=\begin{cases} x^2-x-1 & \text{if } x<3 \\ 2x-1 & \text{if } x>3 \end{cases}$.

i) Determine $\lim_{x\to 3^-} f(x)$ and $\lim_{x\to 3^+} f(x)$.

[***For this part, you must show at least one intermediate step where*** $\lim_{x\to 3^-}$ ***or*** $\lim_{x\to 3^+}$ ***is still needed.***]

ii) Does $\lim_{x\to 3} f(x)$ exist? Give your reason. If it exists, state its value.

iii) Is the function $f(x)$ continuous at $x = 3$? Give your reason for your answer.

[4 marks]

Continued ...

QUESTION 2 [10 MARKS]

a) Use the formal definition of the derivative to compute $f'(3)$ for $f(x) = x(x-2)$.
[***You are reminded to write proper steps.***] [2.5 marks]

Continued ...

b) Find $\frac{dy}{dx}$ for the given functions. [Use ***product rule*** or ***quotient rule*** for differentiation, show proper steps.] [4 marks]

i) $y = (1+\ln x)\sqrt{x}$

ii) $y = \frac{e^{3x}}{1+\sin 2x}$

Continued ...

Continued ...

c) The point (2, 1) lies on the curve $y^2 - 3xy + x^3 = 3$. Use implicit differentiation to obtain $\frac{dy}{dx}$ **in terms of x and y.** Then determine the gradient of the tangent to the curve $y^2 - 3xy + x^3 = 3$ at the point (2, 1). [3.5 marks]

Continued ...

QUESTION 3 [10 MARKS]

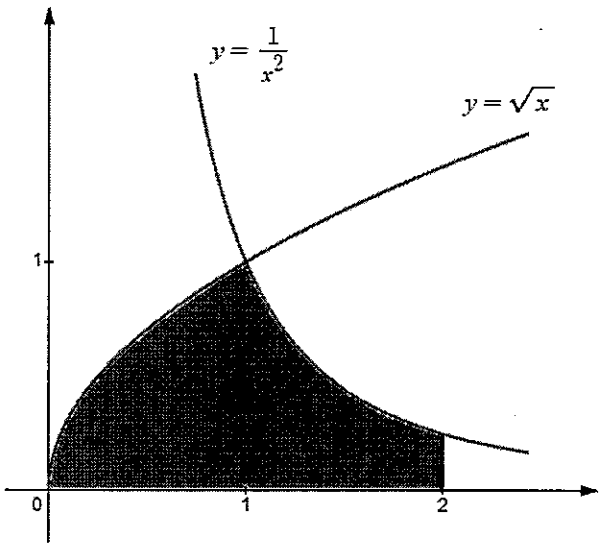
a) Express $\frac{3x+5}{5x^2-4x-1}$ as **partial fraction** in the form of $\frac{A}{5x+1}+\frac{B}{x-1}$. Hence, or otherwise, find $\int \frac{3x+5}{5x^2-4x-1}\,dx$. [3 marks]

Continued ...

b) By using **integration by parts**, find $\int_1^e x \ln x dx$. [3 marks]

Continued ...

c) The diagram below shows the region bounded by the curves $y = \frac{1}{x^2}$, $y = \sqrt{x}$, $x = 2$ and x-axis. Find the area of the shaded region. [4 marks]



Continued ...

QUESTION 4 [10 MARKS]

a) Use the **direct comparison test** to determine whether $\sum_{n=1}^{\infty} \frac{6n^2+1}{3n^3-1}$ converges.

[3 marks]

b) Determine whether $\sum_{n=1}^{\infty} \frac{3^n+4^n}{12^n}$ converges. If it does, find its sum.

[*Note:* $\mathbf{12^n = (3 \cdot 4)^n}$] [3 marks]

Continued ...

c) Let $f(x) = e^x + \sin x$. Obtain the **Maclaurin series** of $f(x)$ up to order 3.
[4 marks]

Continued ...

QUESTION 5 [10 MARKS]

a) Given the function $F(x,y) = e^{-3y}\cos 3x$. Show that $\frac{\partial^2 F}{\partial x^2} = 3\frac{\partial F}{\partial y}$. [3 marks]

Continued ...

b) Given the second order of non-homogenous differential equation subject to initial conditions:

$$y'' + y = 2e^{-x}; \quad y(0) = 0, \quad y'(0) = 0$$

i) Find the complementary solution corresponding to the homogenous equation y_h. [2 marks]

ii) By the method of undetermined coefficient, find the particular solution y_p of the original differential equation above. [2 marks]

iii) Hence, solve the differential equation subject to the indicated initial conditions. [3 marks]

Continued ...

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